Understanding The Pile Stress Wave

What the PDA test shows

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PDA is a commonly used test which shows exactly what is happening as a pile is being driven. It is cheap and quick to set up and shows a vast amount of information about the driving hammer and soil resistance in real time as the pile is being driven.

However, the PDA test has earned a bad reputation because of poor quality interpretations by incompetent operators, compounded by the fact that very few civil engineers in charge of the project understand adequately how to read the PDA results.

This presentation is for engineers to understand the basics of what is the PDA test and what the shape of the stress wave tells us about the pile driving system.

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Introduction

- PDA is a powerful tool to observe and measure what is occurring during pile driving. It can tell us what is happening when the hammer strikes the pile, but also more importantly, it can help us to evaluate the soil capacity and the distribution of soil resistance as well as pile integrity along the entire length of the pile.

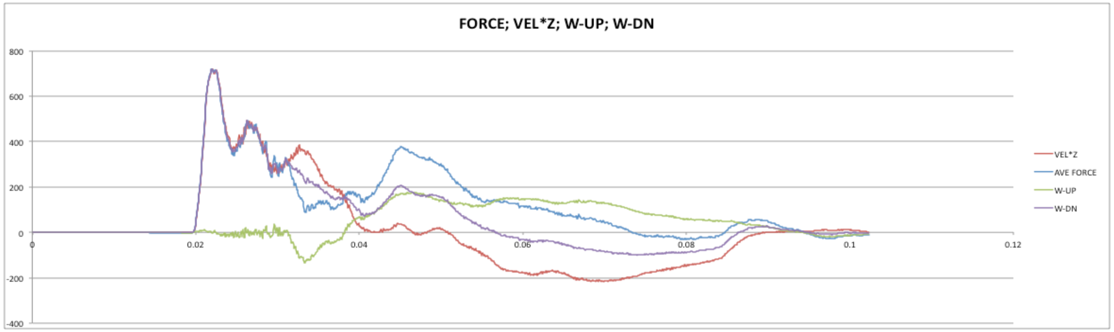
- Compared to a full static load test, PDAs are many times cheaper and more convenient. Therefore, PDAs can be done on a large number of piles (up to 100% of the driven piles in some projects, giving the best insurance against pile failure).

- For driven piles, PDA provides instantaneous information on what is happening during driving, and allows immediate rectification if the pile is not able to achieve its required capacity.

- PDAs can be done on non-driven piles, but requires the pile to be driven for the PDA test to mobilize soil resistance.

- For raked piles, the soil resistance may be very different from vertical piles, and this cannot be seen from vertical borehole data or vertical pile load tests. PDA can measure this effect easily.

- In order to derive full benefits from the PDA, we need to study the different portions of the stress wave, and understand what is causing the changes in shape of the stress wave. This article provides a guide to interpreting what the PDA test data is telling us.



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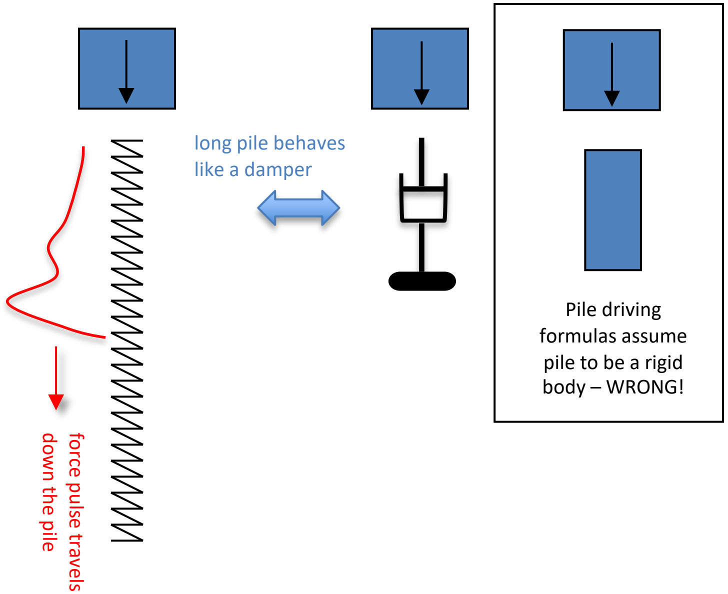
Wave Theory

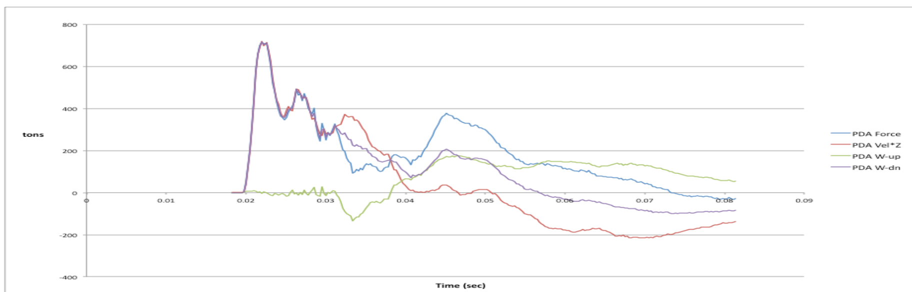
- During pile driving, the **pile behaves like a long elastic spring**. The impact from the hammer does not act on the entire pile instantaneously, but rather takes a finite time to travel down the pile.
- The impact creates a force pulse which travels down the pile at a speed $c = \sqrt{(E/\rho)}$ = 5122m/s for steel. Behaviour of the force pulse is governed by **one dimensional wave theory**.
- A long pile actually behaves like a damper, rather than a rigid body. . A rigid body behaves completely differently, and in certain respects does the opposite of what the correct theory shows.
- **Conventional pile driving formulas** such as the Hiley formula consider the pile as a rigid body and **are** thus **WRONG**. A study by ASCE over a period of 10 years* to find the best pile driving formula found all of them to be as good as useless. Therefore, **conventional pile driving formulas SHOULD NOT BE USED to predict pile capacity**, even for estimation purposes, as they give the wrong trends as well as the wrong answers.
- PDA measures the wave pulse and the reflections caused by soil resistance during pile driving and is a much more correct way of predicting pile capacity.

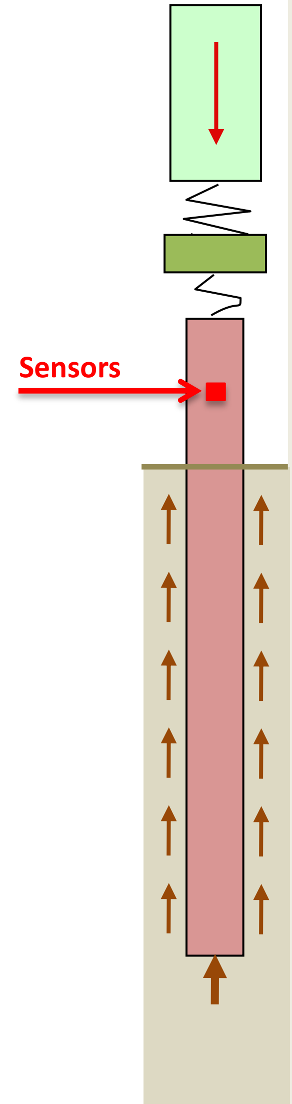


*Likins, Fellenius & Holtz, (2012) "Pile Driving Formulas", *PILEDRIVER, Q2 2012 | Vol. 9, No.2*

PDA Instrumentation

- <u>**PDA**</u> **instrumentation consists of (2)strain and (2)acceleration sensors mounted on the pile.** The sensors are located usually below the hammer and above ground level.
  - Strain and acceleration are measured and converted into force and velocity for analysis.
  - The sensors are usually mounted on an exposed portion of the pile not less than 2D from the top. Two sets of sensors are used; mounted on opposite sides of the pile and averaged in order to isolate the effects of bending of the pile.





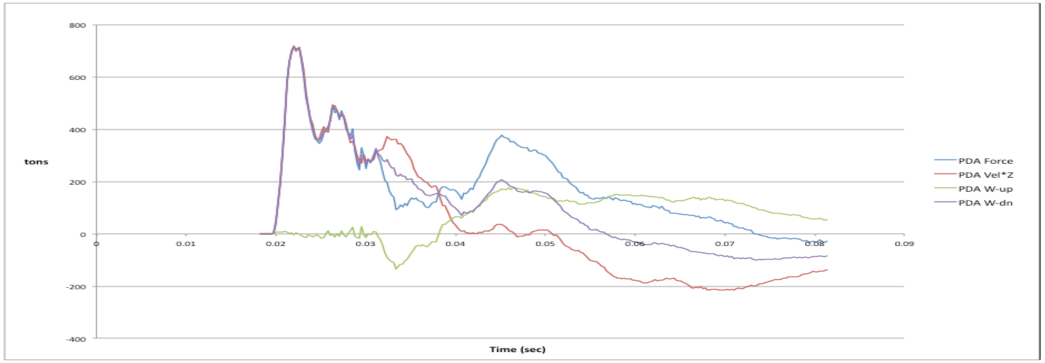
- Output from the sensors can be sent wirelessly to a nearby PC for analysis or even directly over the internet to an engineer sitting in his office.

- Just by looking at the shape of the stress wave, much information can be deduced about the effectiveness of the pile driving equipment, the magnitude and distribution of skin friction end bearing, as well as the integrity of the pile. Therefore, it is useful to learn about what exactly the wave shape is revealing.

- Soil resistance during driving can be estimated directly on site from the PDA output using the **Case Method**, but this is inaccurate. To properly obtain the soil capacity, it is necessary to do a **wave matching** analysis such as **CAPWAP, iCAP**. ALC uses **Full Wave Matching** (FWM) which gives all the pile driving parameters as well as the soil parameters.

Force, Velocity, Wup and Wdn

- The **PDA** sensors measure **strain** and **acceleration**, which translate to **force** and **velocity** respectively. The velocity is multiplied by the impedance Z of the pile to give the result in force units. These are then plotted against time. (blue – Force; red – Vel*Z)



- It is often more convenient to work with the **downward and upward components of the stress wave, Wdn and Wup**.
  - The **downward wave, Wdn = (Force + Vel*Z) / 2** is generated by the hammer. It is convenient to remember the downward wave as the average of the force and velocity traces.
  - The **upward wave, Wup = (Force – Vel*Z) / 2** is the reflected wave due to soil resistances and changes in pile cross-section. The upward wave is the difference between the force and velocity traces divided by 2.

- Wdn (brown) and Wup (green) are calculated from the PDA Force (blue) and Velocity (red) data and can be plotted together on the same graph for easy reference.

- According to wave theory, Wdn and Wup travel in opposite directions and do not affect one another.

- Force and Velocity can also be back-calculated from the Wdn and Wup waves:-
  - Force = Wdn + Wup
  - Vel*Z = Wdn - Wup

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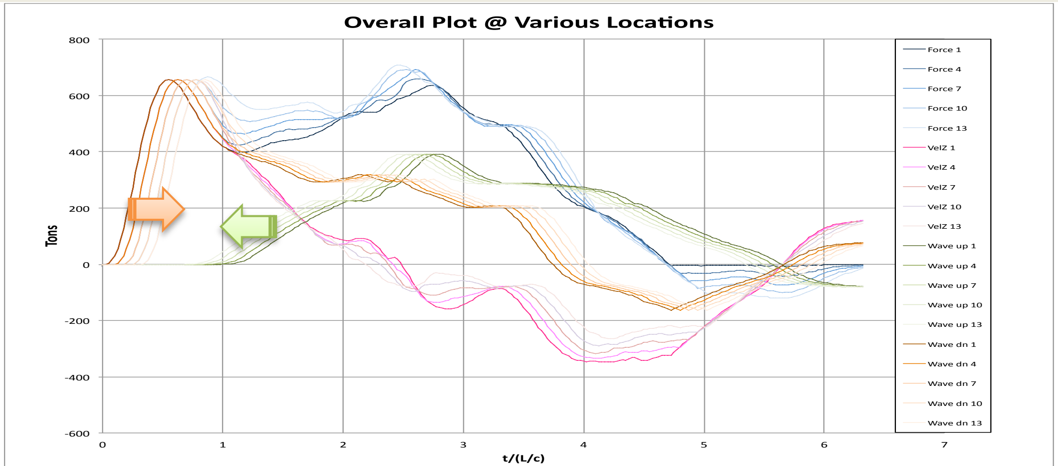
Wave Movements in the Pile

- A downward stress wave is generated by the impact of the hammer ram on the cushion. The wave passes through the cap block and is transmitted down the pile.
- The downward wave is modified and also partially reflected upwards by the soil resistance and changes in pile cross-section along the pile.
- When the downward wave reaches the bottom end of the pile (at t = L/c), it reverses direction and becomes an upward wave. The amount and type of reflection depends on the soil condition at the tip of the pile.
- The upward wave then travels up the pile and becomes a downward wave again when it reaches the top end of the pile (at t = 2L/c). On the way up, it is again modified and partially reflected downwards by the soil resistance along the pile.

- By analyzing the shape and magnitude of the wave reflections, we can deduce the pile cross section, the soil resistance and pile integrity at each point along the pile.

Effect of Sensor Location

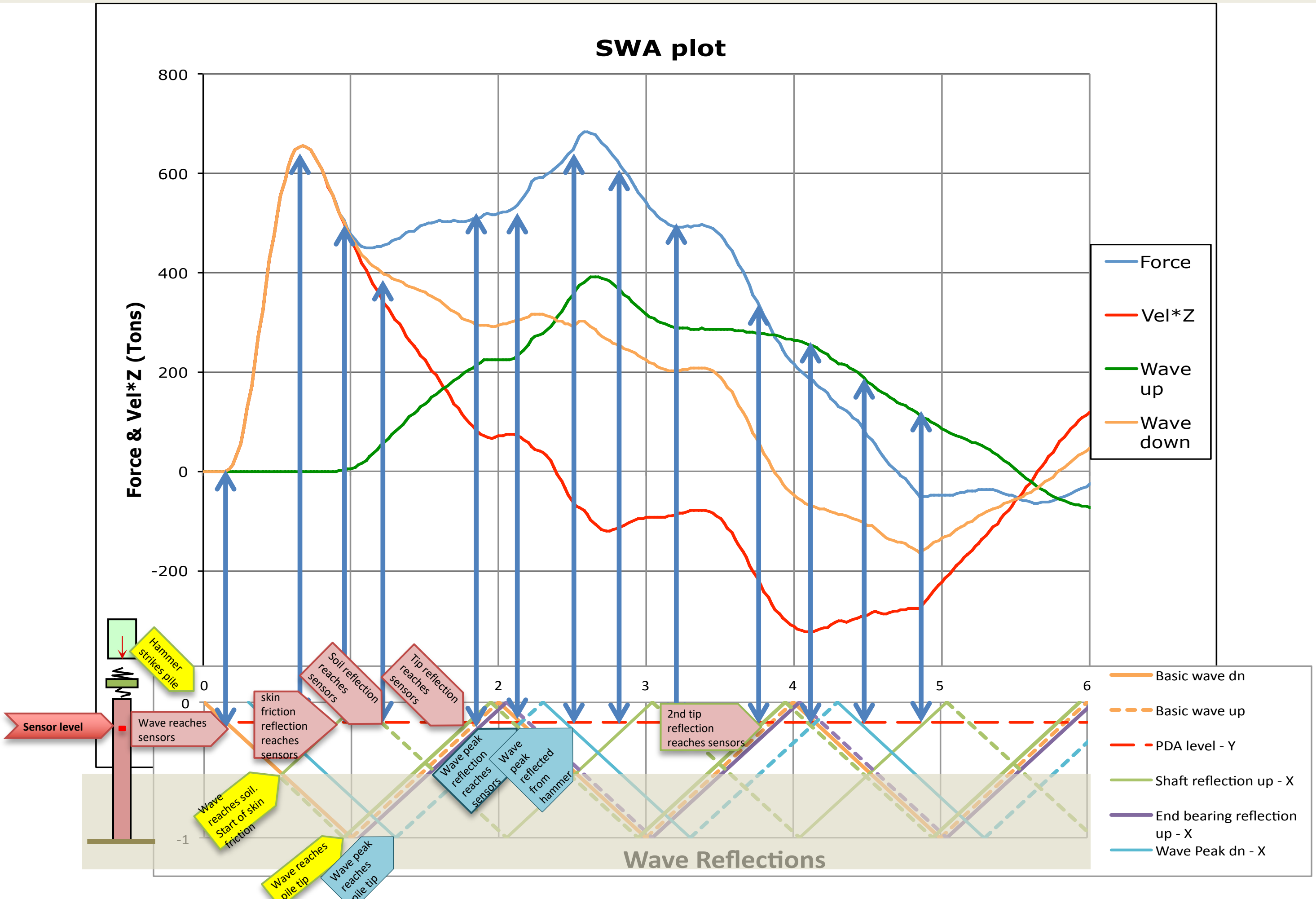


This plot shows the effect of placing the sensors at different locations along the pile. The darkest lines are for the sensors located at the pile top, and the progressively lighter lines are for sensors located further and further down the pile, but still above the mud line.

- **The WaveDown and WaveUp curves (brown & green) do not change in shape with different sensor locations. They only are shifted to the right and left side respectively (phase shift in time).**
  - The Force and Velocity curves (blue & red) do change shape as the are the sum and difference respectively of the WaveDown and WaveUp curves which are shifted in opposite directions.
  - This makes it easier to work using the WaveDown and WaveUp curves, rather than using Force and Velocity.

- The lower down the sensor is, the longer the lag time before the **WaveDown pulse** arrives at the sensor, thus **shifting the curve to the right**. The amount of shift is the time it takes for the wave to travel from the pile top to the sensor location.
- The **WaveUp pulse** travelling in the opposite direction will reach the lower sensor earlier, hence the **shift of the curve to the left** by the time required for the wave to travel between the sensor and pile top. **This time shift must be accounted for to accurately locate the pile tip reflection.**

Wave Reflections



A study of the wave reflections is key to understanding the stress wave. Significant changes in the wave shape occur when the Fdown or Fup wave passes through the PDA measurement level shown by the red dashed line.

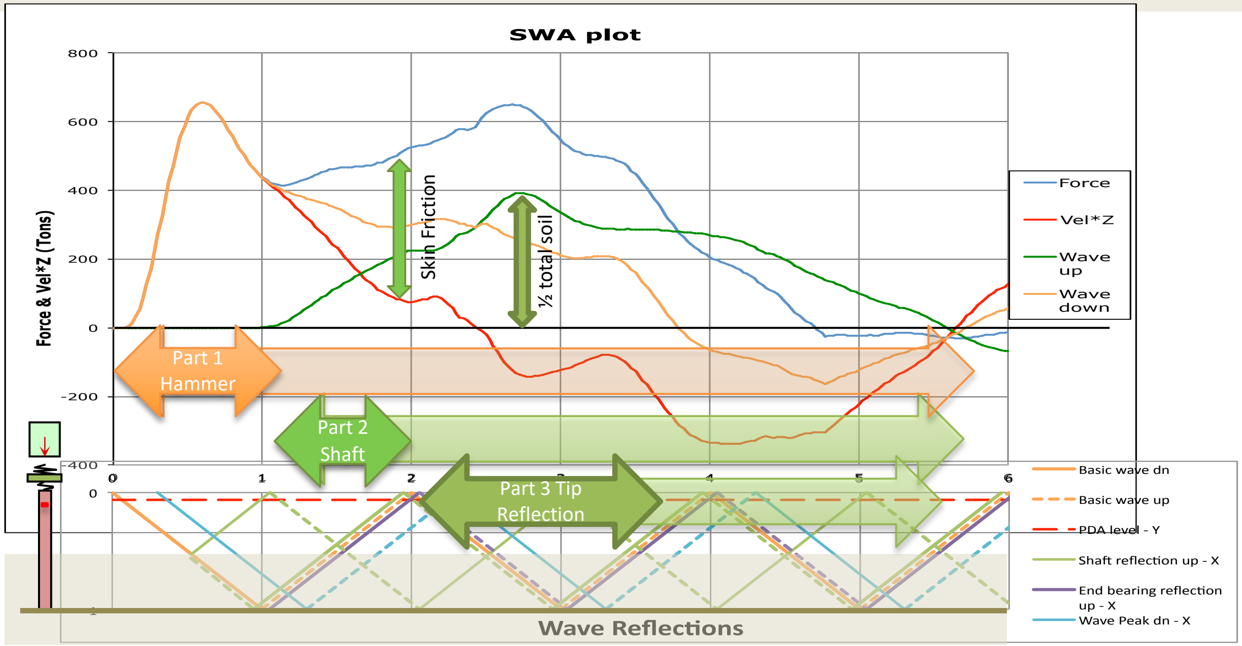
3 PARTS OF THE STRESS WAVE

Part 1 – Hammer

Part 2 – Shaft

Part 3 – Tip Reflection

3 Parts of the Stress Wave



In interpreting the stress wave the 3 parts of the stress wave are:-

- **Part 1 – Hammer**

  WaveDown is affected only by the hammer and pile top properties. Not yet affected by soil.

  The downward wave shows the characteristics of the hammer and pile top.

- **Part 2 – Shaft**

  WaveUp is modified by the skin friction along the side of the pile and by changes in pile cross section.

  The difference in separation of the F and V curves gives the soil skin friction.

- **Part 3 – Tip**

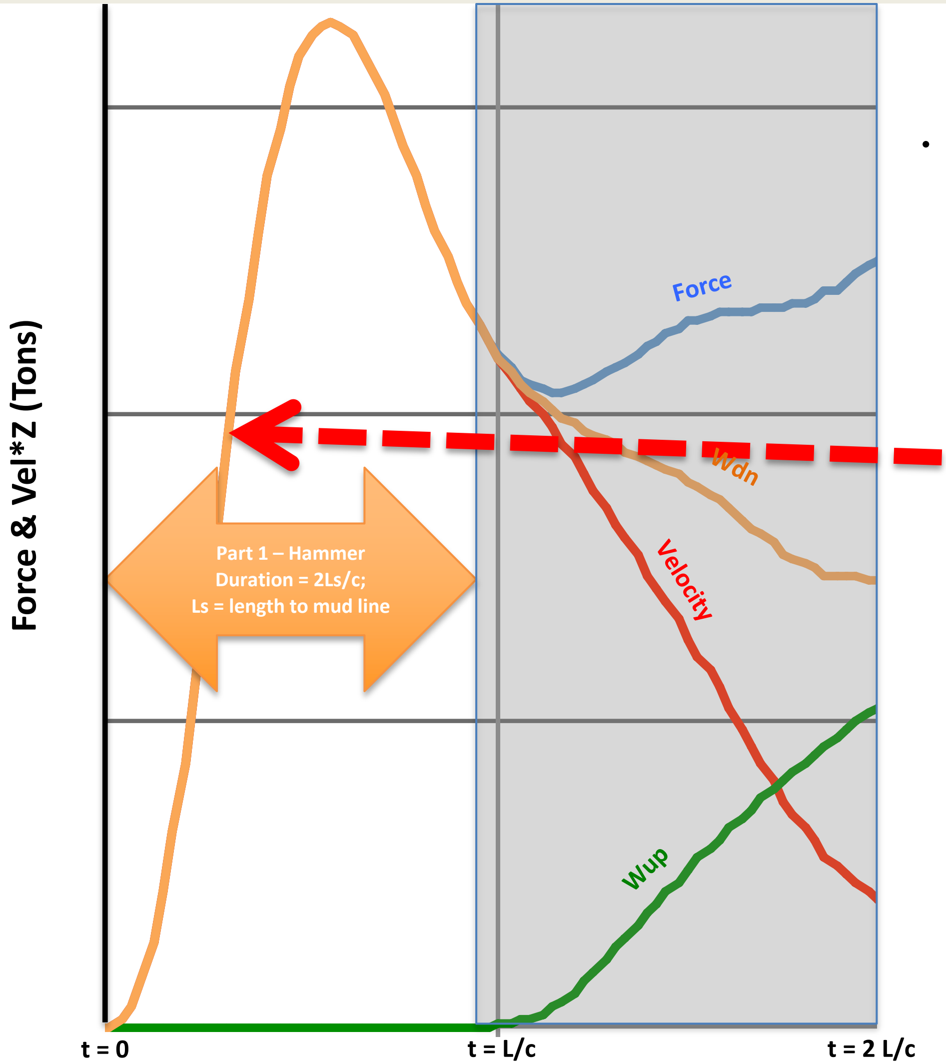
  WaveUp is additionally modified by the end bearing resistance as well as the pile tip reflection.

  The maximum height of the WaveUp curve equals half the total soil resistance.

Part 1 of the stress wave

HAMMER

Part 1 - Hammer

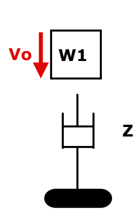


- The first part of the downward wave, before the wave reaches the mud line, is only affected by the hammer and pile top characteristics. Therefore, this portion of the downward wave can be used to determine the pile driving parameters, i.e. the ram weight, the cushion stiffness and the helmet weight.

At the initial part of the wave, the **Force, Velocity and Wave Down curves should coincide**. If they do not, the results are not properly calibrated and should be rejected.

This is called **proportionality**.

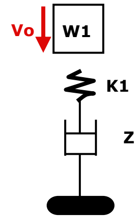
Effect of Hammer Model Complexity



Hammer Model (1)

Direct impact of ram on pile
W1 = 20.5 T
Vo = 3.7 m/s
Z = 182 T.s/m

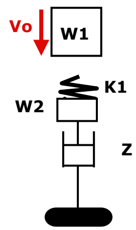
- Simple theoretical formula.
- Results in an exponentially decaying stress wave



Hammer Model (2)

Ram + cushion
K1 = 200 T/mm

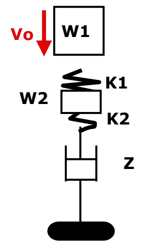
- More complicated theoretical formula.
- Rounds off the exponential curve.
- Still does not look like the PDA data



Hammer Model (3)

Ram + cushion + helmet
W2 = 2.2 T

- Very complicated theoretical formula.
- Shape looks more realistic.
- Still insufficient to match actual measured wave

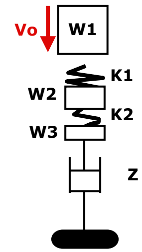


Hammer Model (4)

- **BEST MODEL**

Ram + cushion + helmet + pile top spring
K2 = 300 T/mm

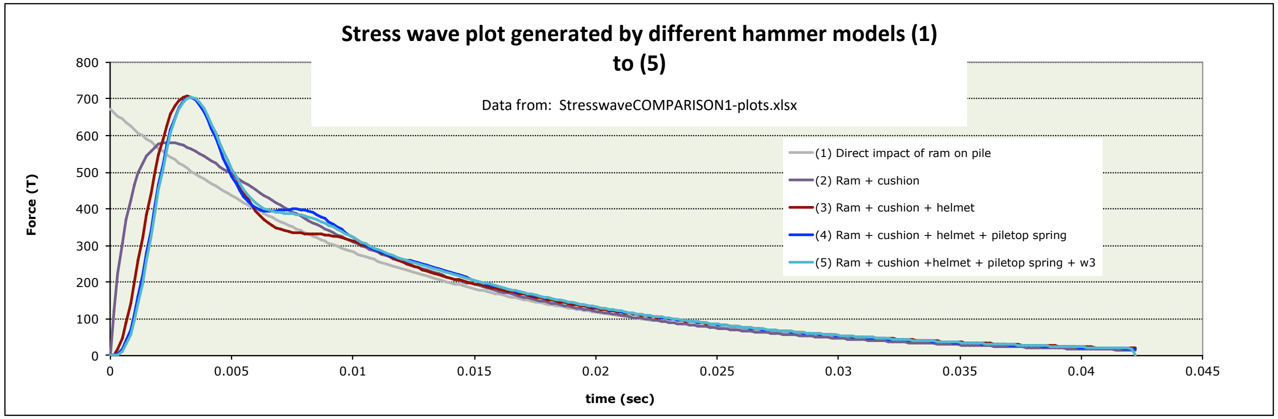
- Too complex to use theoretical formulas,
- Can be simulated by mass-spring computer model.
- Gives a good fit with measured stress wave



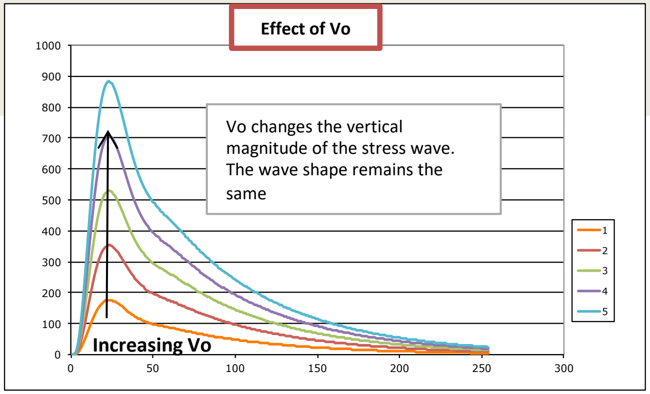
Hammer Model (5)

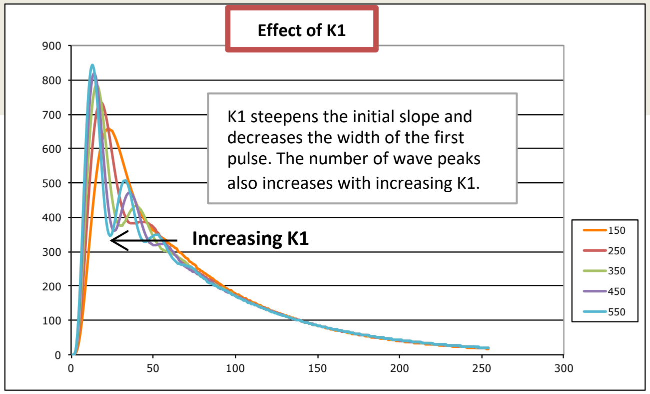
Ram + cushion + helmet + K2 + pile top weight
W3 = 500 kg

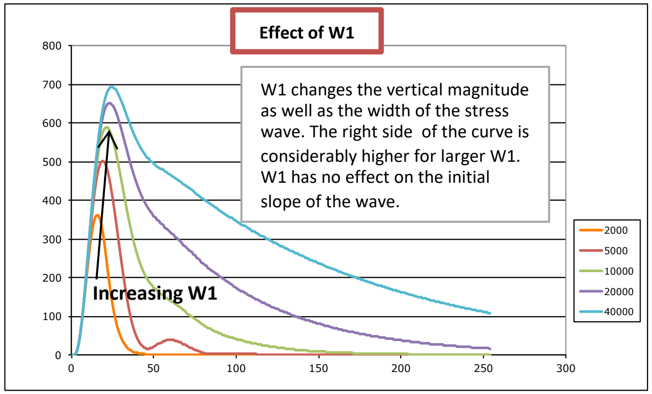
- Added weight W3 on pile top for research purposes.
- Considered not necessary as model(4) is sufficient to represent the hammer.

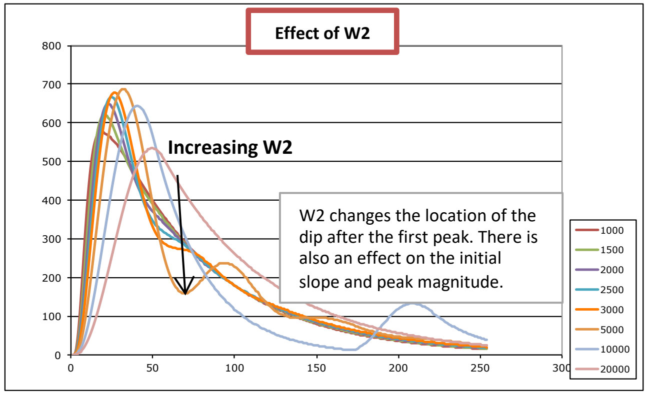


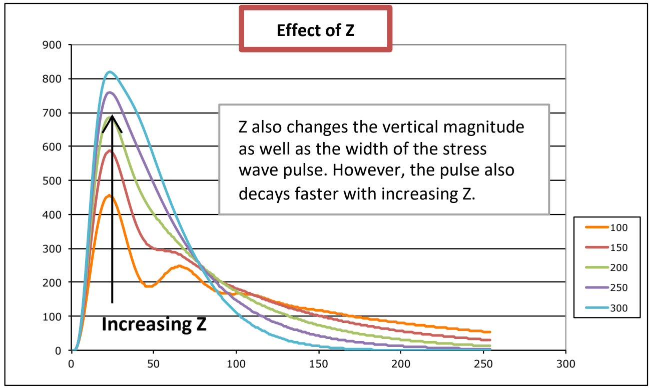
Effect of Hammer Parameters

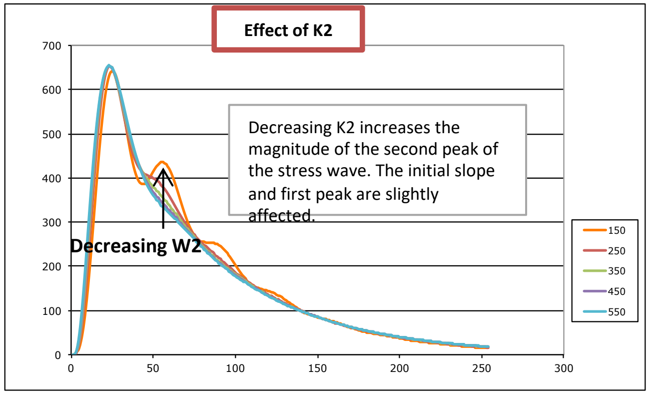


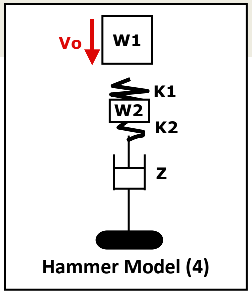












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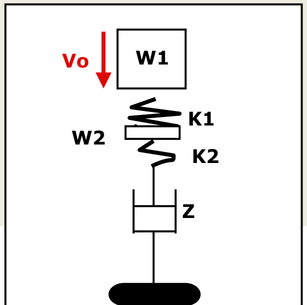
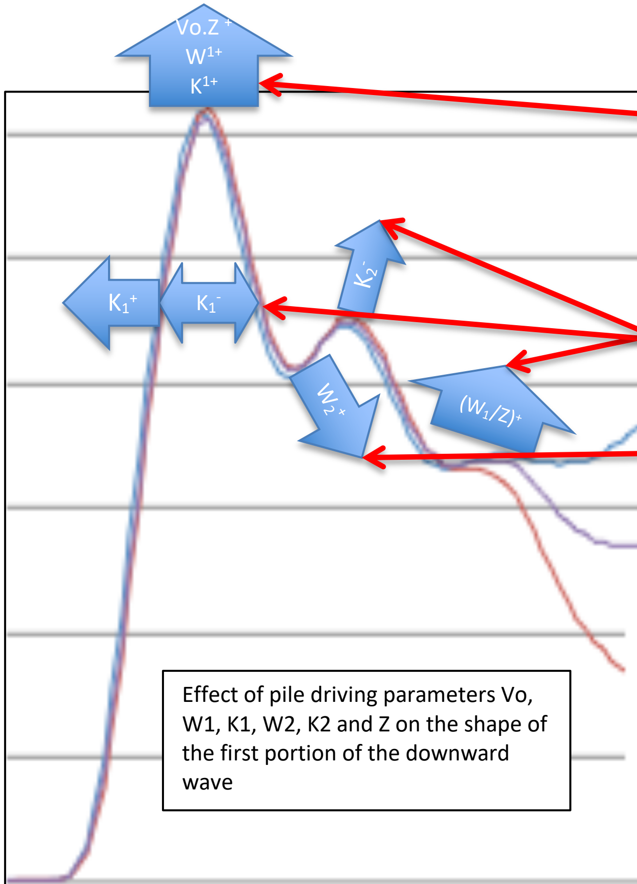
Matching Hammer Characteristics



Hammer Model (4)



Effect of pile driving parameters Vo, W1, K1, W2, K2 and Z on the shape of the first portion of the downward wave

In wave matching, we attempt to recreate the stress wave measured in a PDA field test by changing the parameters in the computer model. The match of pile driving parameters is actually not difficult to achieve as the first part of the downward wave is only affected by the hammer and pile top characteristics.

The following guidelines have been derived from experience to fit the computed downward wave to the measured one:-

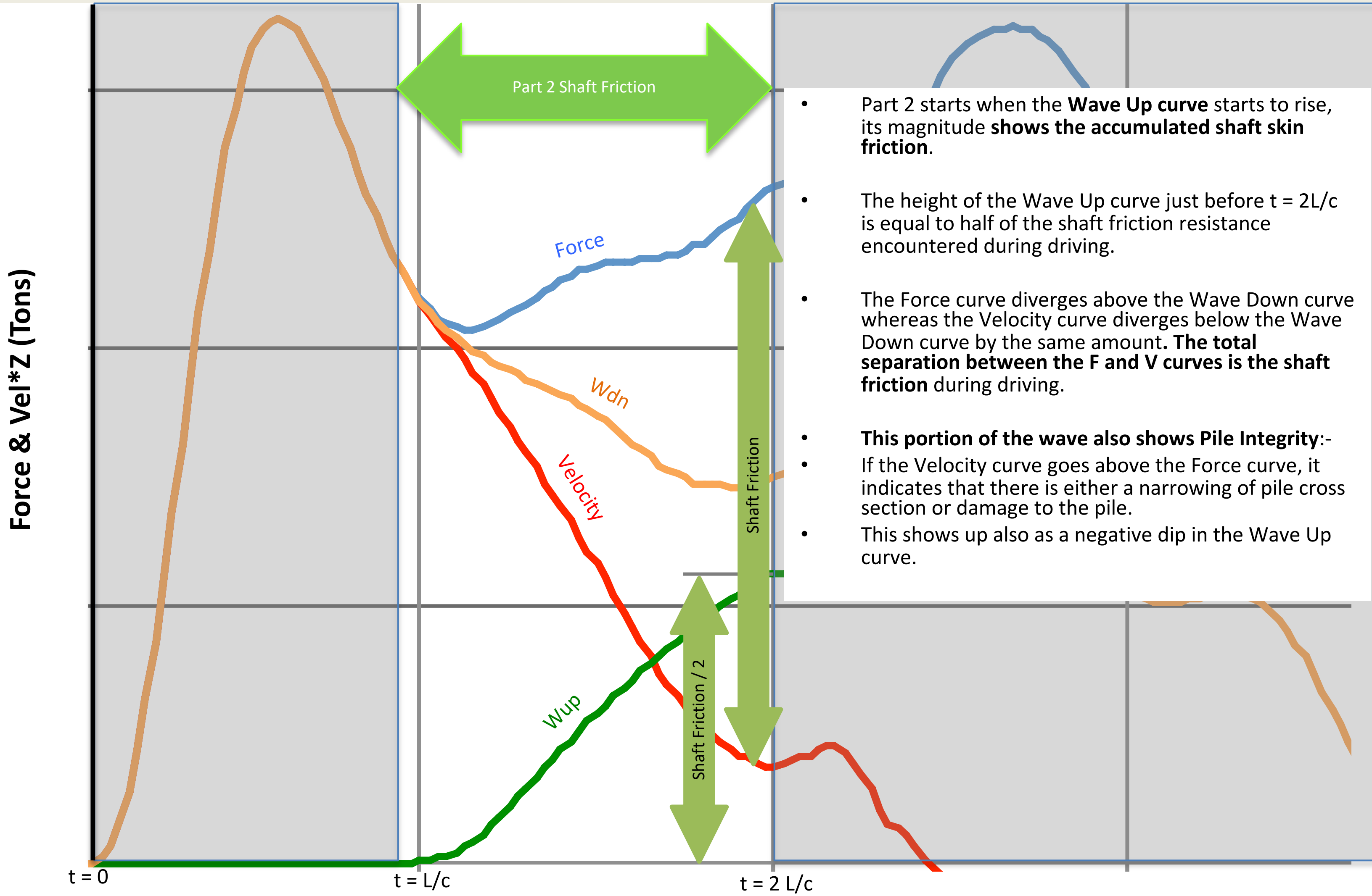
1. The height of the force wave is affected mainly by Vo.Z and W_1. It is also affected by K_1 to a lesser extent
2. The ratio Z/W_1 i.e. the pile impedance / the ram weight determines the rate of decay of the function, i.e. how much it slopes downward.
3. K_1 determines the slope and width of the initial peak.
4. W_2 determines how far the first dip is from the first peak.
5. K_2 determines the relative height of the second peak. May not have second peak with soft K_1.

- These 5 rules allow us to understand the hammer impact conditions and to fit the initial shape of the downward force wave reasonably well in most cases.
- Note the above are the major effects. Each parameter also affects other parts of the curve to a lesser degree.

Part 2 of the stress wave

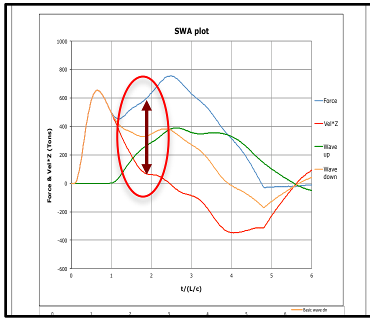
SHAFT FRICTION

Part 2 – shaft friction

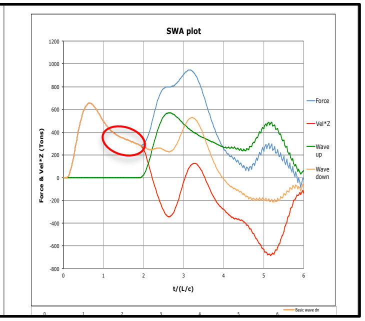


- Part 2 starts when the **Wave Up curve** starts to rise, its magnitude **shows the accumulated shaft skin friction**.
- The height of the Wave Up curve just before t = 2L/c is equal to half of the shaft friction resistance encountered during driving.
- The Force curve diverges above the Wave Down curve whereas the Velocity curve diverges below the Wave Down curve by the same amount**. The total separation between the F and V curves is the shaft friction** during driving.
- **This portion of the wave also shows Pile Integrity**:-
- If the Velocity curve goes above the Force curve, it indicates that there is either a narrowing of pile cross section or damage to the pile.
- This shows up also as a negative dip in the Wave Up curve.

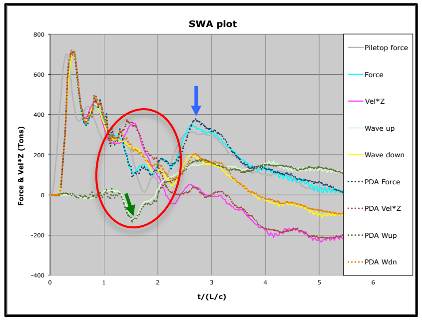
Examples



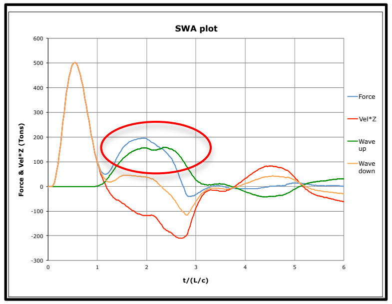
- **Good skin friction**
- F and V curves far apart at t = 2L/c



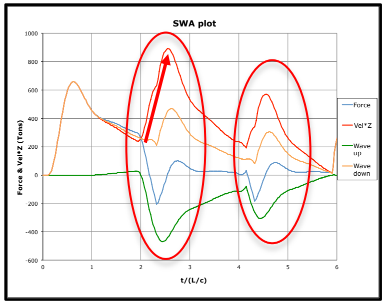
- **No skin friction**
- F and V curves close together before t = 2L/c



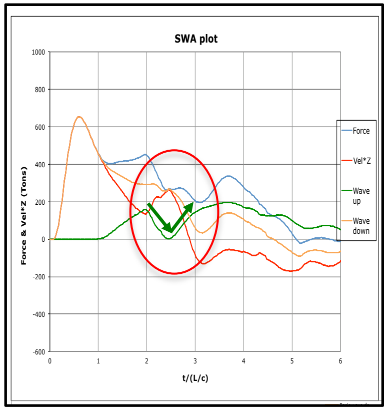
- Thin pile section substituted.
- V rises above F and –ve Wup.
- F peak after 2L/c is very low, => poor drivability



- **Hammer too small**
- F peak after 2L/c is much lower than first peak



- **Pile has not set**
- very little soil resistance
- strong reflection at pile toe (V >> F)



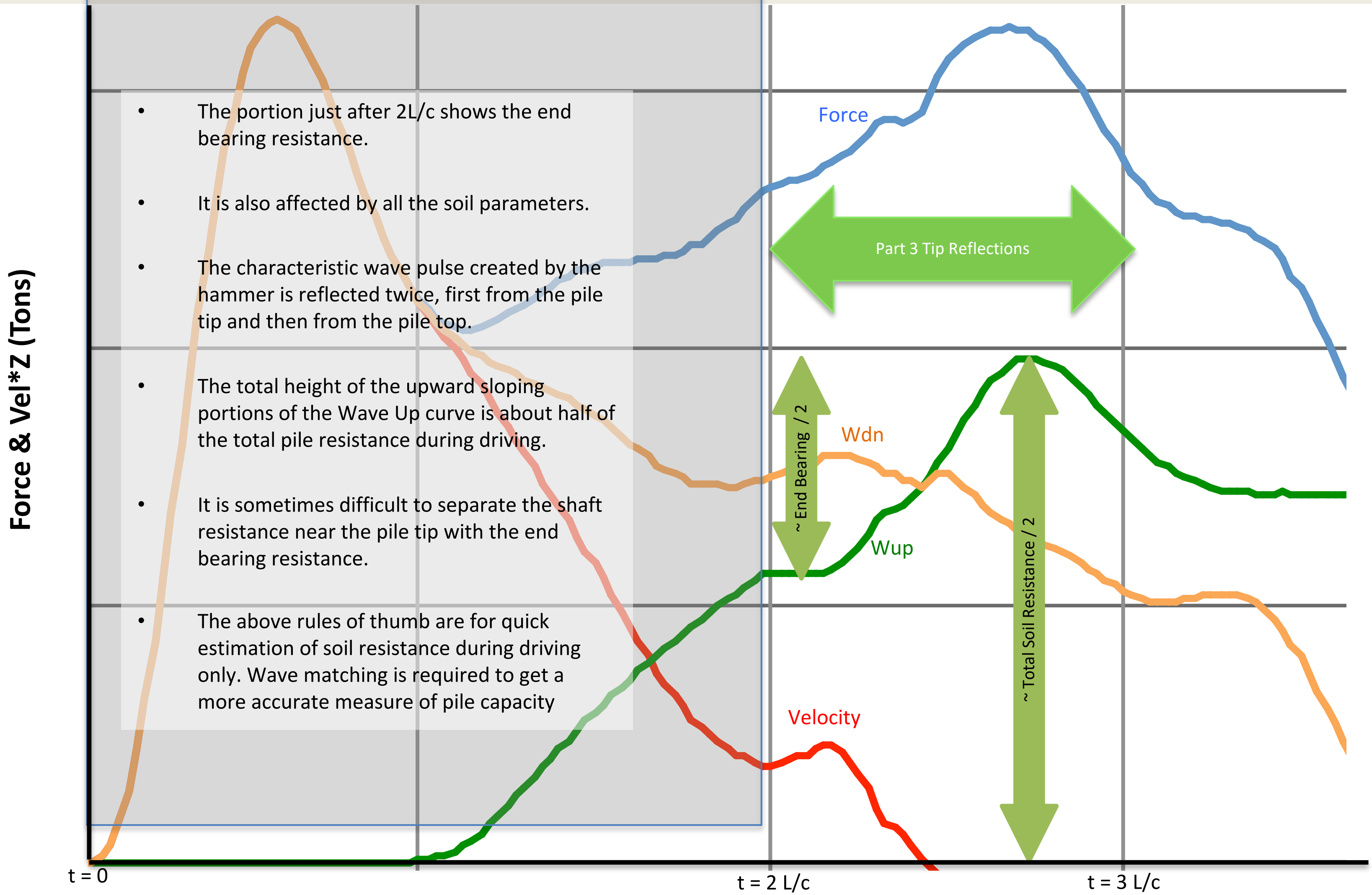
- **Soft base = large Qt**
- **Wup** has a double dip
- some reflection at pile toe

Part 3

TIP REFLECTION

Part 3 – End Bearing

- The portion just after 2L/c shows the end bearing resistance.
- It is also affected by all the soil parameters.
- The characteristic wave pulse created by the hammer is reflected twice, first from the pile tip and then from the pile top.
- The total height of the upward sloping portions of the Wave Up curve is about half of the total pile resistance during driving.
- It is sometimes difficult to separate the shaft resistance near the pile tip with the end bearing resistance.
- The above rules of thumb are for quick estimation of soil resistance during driving only. Wave matching is required to get a more accurate measure of pile capacity



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Soil Resistance During / After Driving

- PDA records the actual instantaneous soil resistance during driving. This may be very different from the soil resistance available long-term to support the design pile load.

- There are 2 components to compensate for:-
  1. the transient dynamic soil resistance, as opposed to the static resistance, during driving.
     - There are procedures to calculate this in the computer simulations
  2. **Soil Setup**, which is the increase in skin friction over time
     - In certain types of soil, it may even decrease, but this is rare.

- To **measure soil setup**, the usual procedure is to perform a **second PDA test** a few days after the pile is driven. The soil resistance is measured for the first few blows before the soil has time to soften. This **re-strike test** when compared with the results of the original PDA test will show the change of soil resistance over time.

- It is important to do a re-strike test on a number of selected piles to determine the effect of soil setup.

Using the computer for

WAVE MATCHING

- Wave matching is using the computer model to recreate the PDA test results.
- The hammer, pile and soil parameters are varied in the computer model such that the computer generated stress wave match the actual PDA measurements.

Computer Modeling

- Computer programs for pile driving analysis using the wave equation have been around since the 1960s. The pioneering work was done by E.A.L. Smith[1][2], who proposed modeling the pile as a series of lumped masses and springs. Smith also formulated computer model representations for the pile hammer and cushion, as well as the soil model taking into account the soil quake Q and damping parameter J.

- The Method of Characteristics (MOC) was developed by Van Hamme et al[7] in the 1970s as an alternative method of calculating the wave propagation down the pile. This method represents the pile as a transmitting conduit with impedance Z and tracks what happens to the stress wave when it encounters soil resistance or changes in pile impedance.

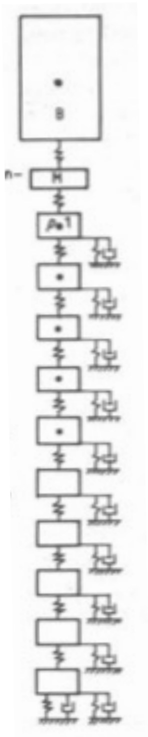
- Traditionally, there are 3 parts to pile driving analysis:-
  1. Before the start of piling, a **WEAP** (Wave Equation Analysis of Pile Driving) program is used to predict the effectiveness of a proposed pile driving hammer. This commonly uses a Smith model of lumped masses and springs.
  2. During piling, a **PDA** (Pile Driving Analyzer) test, consisting of strain and accelerometer instrumentation to measure the pile stress wave in real time. These measurements can be used to roughly estimate the pile capacity using the **CASE** method.
  3. A more accurate assessment of pile capacity is made afterwards in the engineering office using a **wave matching** program such as **CAPWAP or iCAP**, which uses the Method of Characteristics model to calculate the effect of changing various soil parameters along the pile on the stress wave.

- In 2004, ALC developed a spreadsheet implementation of the Smith model. The spreadsheet interface was considered to be the most expeditious for development and visualization of the results. Experiments in full wave matching led to a hybrid model using lumped mass springs to represent the hammer and the Method of Characteristics to track the stress wave in the pile.

- As a result, WEAP, PDA analysis and CAPWAP can all be done by a single spreadsheet program. The use of the spreadsheet makes wave matching, plotting and comparison of results and changes to the computer model easy and convenient.

Methods of Calculating Stress Wave 1

- **Smith method – used in PWA.xls**



– The Smith method simulates the pile as a series of masses and springs. The original Smith model has been surprisingly good at simulating the stress waves measured in the PDA tests.

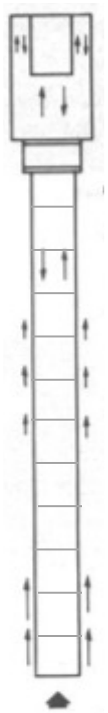
– However, the Smith method has the disadvantage that the force is calculated at the spring location, while the velocity is at the mass location, which is not the same point. (half a segment length difference)

– Since PDA signals are taken at the same point in the pile, there is an inherent inaccuracy in the Smith simulation when used for wave matching.

– Simulating the pile as a series of lumped masses and springs is not as direct and elegant as the second method below, which tracks the transmission of the upward and downward stress waves through the pile.

Methods of Calculating Stress Wave 2

- **Method of Characteristics – used in SWA.xlsx**



- There is another method of wave analysis **– the Method of Characteristics -** which treats the pile and hammer as a continuous media with varying impedance in which the wave is being transmitted.

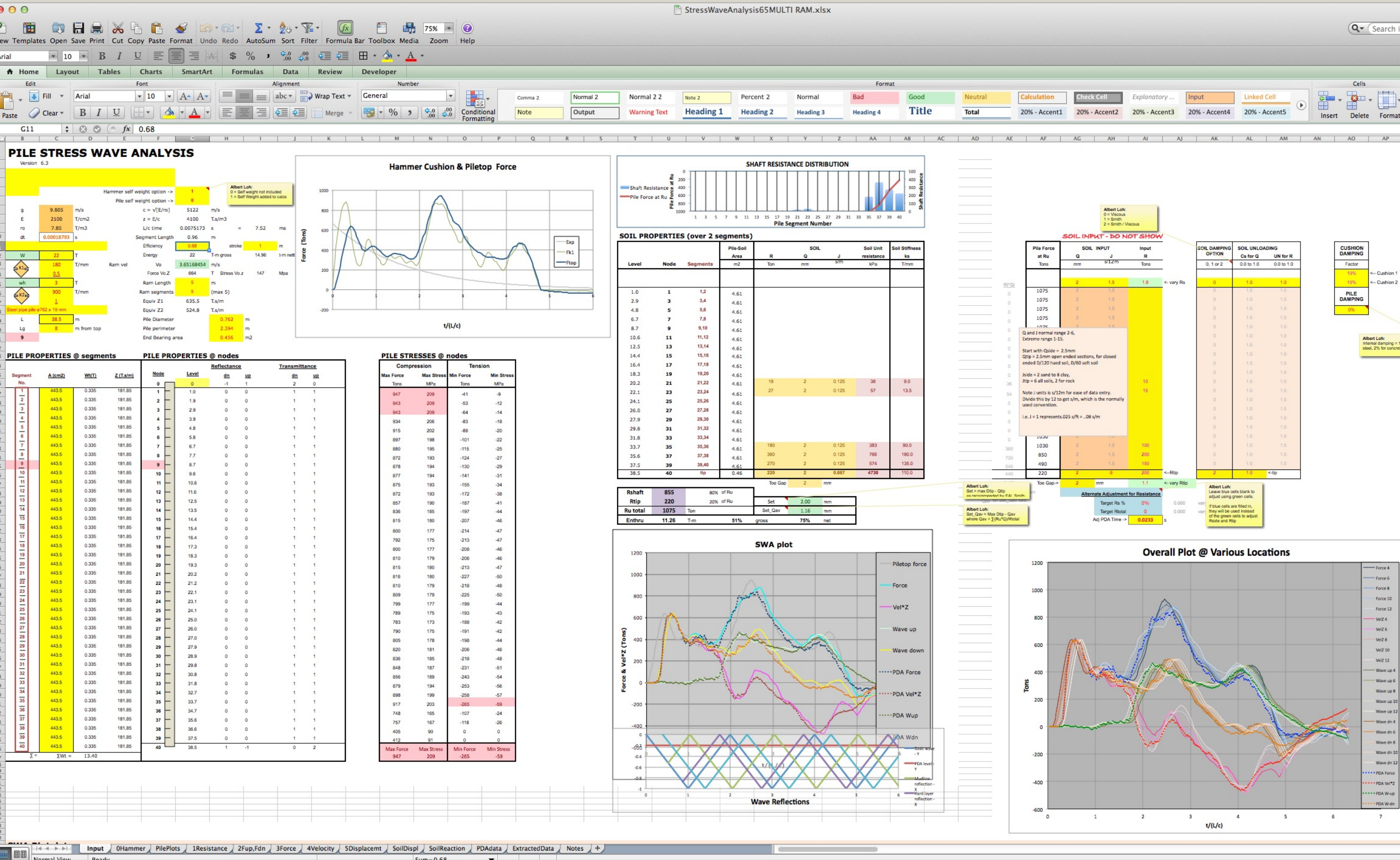
- The stress wave is separated into a downward and an upward travelling component. Each component is propagated independently of the other. **Tracking the up and down waves is the basis of the method of characteristics.** The procedure inherently shows directly how the wave is being transmitted and reflected and helps in understanding what factors affect the pile driving system.

- The hammer and pile are divided into segments, each with uniform cross section, and the soil resistance is assumed to act as concentrated loads in between segments. The upward and downward components of the stress wave are assumed to be transmitted unchanged through the individual segment and is modified by the soil resistance / change in pile section at the junction between segments.

- This method has the advantage that the force and velocity are calculated at the same point in the pile, and is thus better suited for wave matching. It also gives twice the number of data points along the pile compared to the Smith model for the same time interval.

- In the Method of Characteristics, the elements are considered as lengths of conducting medium. The length of each element must be in multiples of the time interval multiplied by the wave propagation velocity. This is not ideal for simulating hammer and cushion parameters. It is much more convenient to represent the cushion as a spring rather than a fictitious piece of material with its length limited to multiples of the time period times the wave speed in that material. Similarly, a helmet is better represented as a point mass rather than a length of steel material.

Spreadsheet Input / Output



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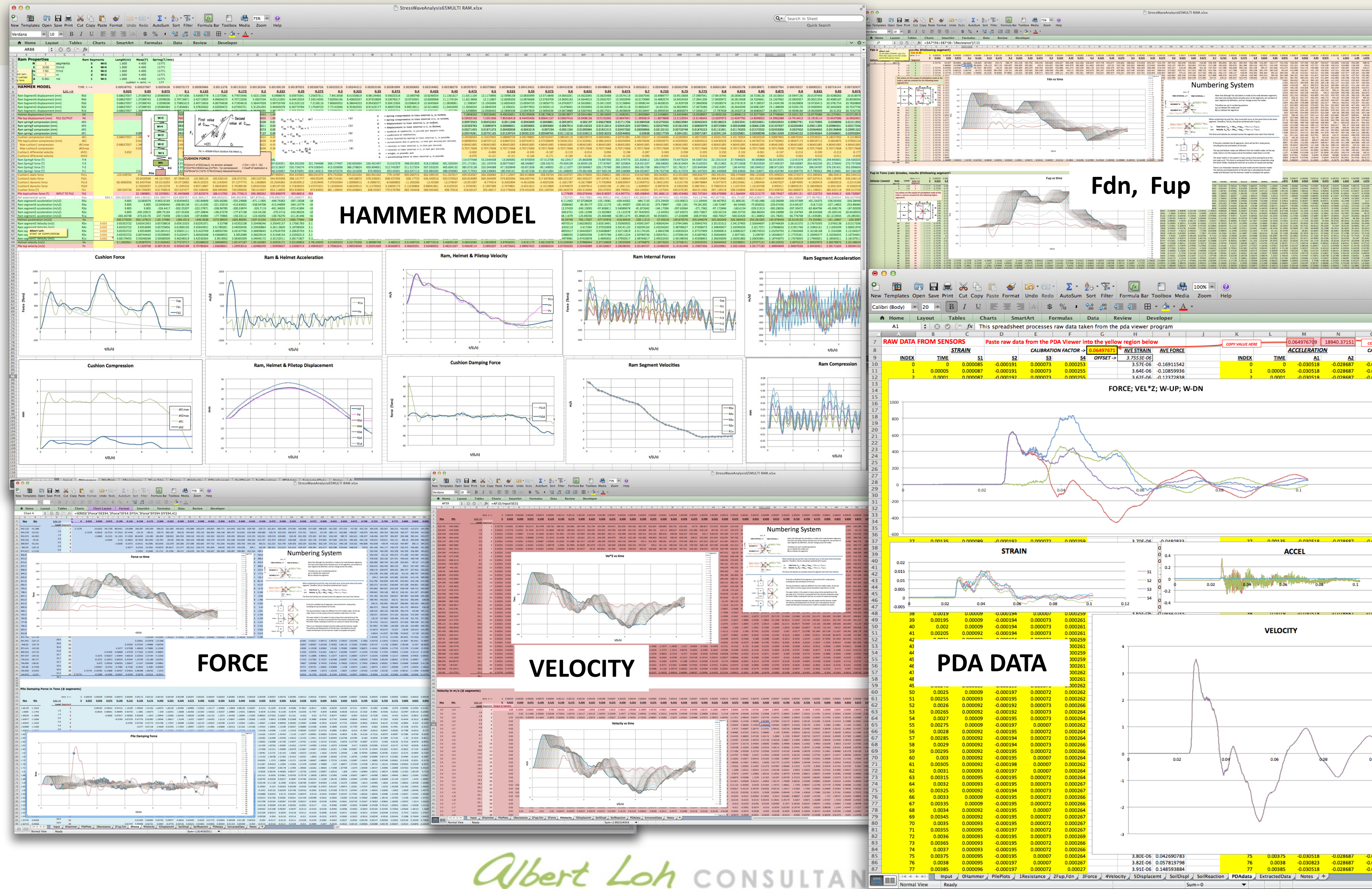
Screenshots



WAVE MATCHING PROCEDURE

Start from left to right. Try to match all the peaks of all the curves, but most importantly, try to match Wup as closely as possible, as this gives the pile capacity. Past 3L/c, the matching becomes less and less important to achieve.

1. Match first impulse wave peak using pile hammer and pile tip parameters.
   - Pile cushion stiffness K1 determines the width of the first pulse.
   - Hammer efficiency and stroke have the same effect of raising the first peak.
   - Weight of hammer W1 tilts the right side of all plots upwards.
   - Weight of helmet W2 moves the position of the second peak to the right.
   - Top of pile spring K2 determines the height of the second peak.
   - The ram length and number of ram segments affects the detail shape of the curve.
   - The coeff of restitution for the cushions e1 and e2 are usually best set between 0.9 to 1.0.
   - Can try different values of e1, e2 as well as changing the cushion damping for better match.

2. MOST IMPORTANT. Match the Wup curve at t < 2L/c using individual shaft resistances to match the PDA Wup plot.
   - Start from the top soil segment and work towards the pile toe.

3. Match the plateau of Wup using end bearing, keeping skin friction constant.

4. Match the peaks near 3L/c using Jshaft to suppress the peaks. May need to readjust the shaft resistance and shaft Q.
   - Shaft J decreases shaft resistance and tip J decreases tip resistance. Not easy to distinguish between the two.

5. Match the Wup peak after 2L/c using Jtip. May need to readjust end bearing and Qtip every time you adjust Jtip.
   - Q side moves the shaft portion of the curves to the right. Qtip moves the Wup curve near 2L/c to the right.

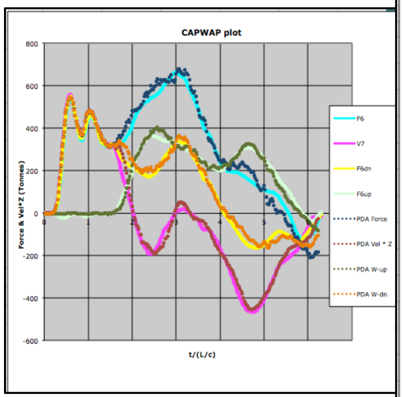
6. PILE SET must be > 0. Can try matching pile set using total soil resistance. This is not always accurate.

7. Soil Unloading R and Q have a small effect on the right side of the chart > 3L/c. Mostly for cosmetics.

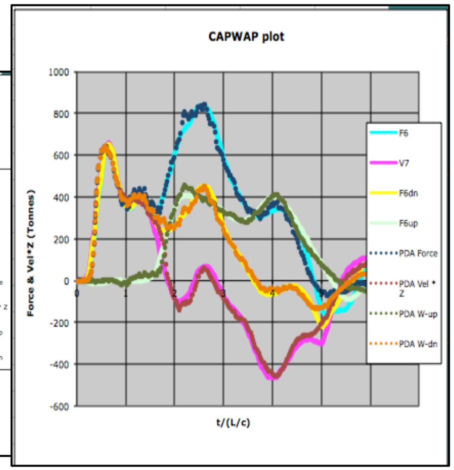
Remember the only parameters that matter are the SHAFT RESISTANCE and the TOTAL RESISTANCE. All other parameters are of little use except for matching the plots in order to get the soil resistances.

Experience with Full Wave Matching

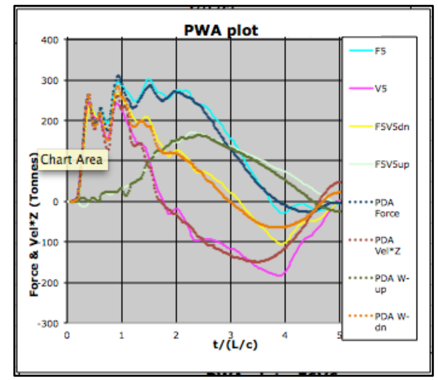
Experience with matching different kinds of wave shapes is shown in these examples.
A reasonable match has been found in all cases analyzed so far, even with widely varying wave shapes.



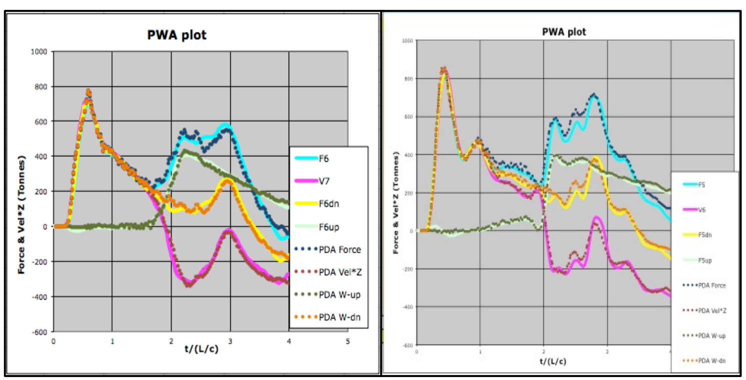
Steel Pipe Pile End of Drive



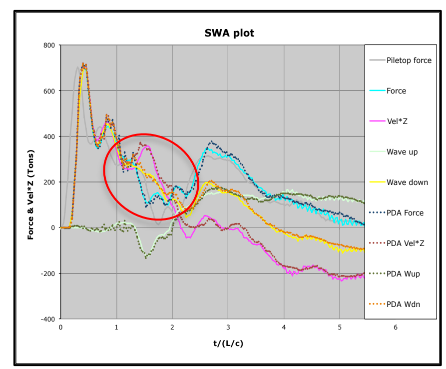
Steel Pipe Pile ReDrive



Concrete Spun Pile Driven With Follower



Different Wave Shapes due to different Hammer/Cushion/Soil
Characteristics for dolphin raker piles



Thin pile section substituted.
Caught red-handed!

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Conclusions

- PDA is like an x-ray insight into the innards of the pile. The shape of the PDA curves shows a great deal of information on the status of the pile driving, and if engineers learn how to read it, it contains much useful information which is not available by other means.
  1. The first part of the PDA wave trace reveals the driving characteristics of the hammer used.
  2. The second part of the PDA trace shows the skin friction encountered by the pile as well as the condition and integrity of the pile itself.
  3. The third portion of the trace shows the pile end bearing capacity and soil reaction characteristics at the pile tip.

- PDA is done in real-time during driving, and thus allows immediate remedial action to be taken to rectify inadequate pile driving or pile damage during driving.

- Boreholes only show the vertical soil profile at one point location, and the number of boreholes is limited. Static load tests are extremely cumbersome and costly, and can only be done in very limited numbers. The low cost of the PDA test allows widespread monitoring on a large number of piles, and shows much more than a conventional load test. In particular, for raked piles PDA shows the effect of the pile rake, which may be different depending on rake direction in certain soil conditions.

- Ideally, we should do a PDA test for every pile, and this has been done on some of our marine projects, where piles are expensive to install and very difficult to do static load tests on. With modern electronics, the test data can be sent directly from the sensors over the interne. Modern software like iCAP can do wave matching automatically and immediately, giving the assurance of the structural and geotechnical capacity of each pile.

‘ A driven pile is a tested pile ‘

THANK YOU

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Smith Method

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PDA, CAPWAP, Analytical Modeling

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5. Rausche, F., Likins, G., Liang, L., and Hussein, M., (2010), "Static and Dynamic Models for CAPWAP Signal Matching", *The Art of Foundation Engineering Practice,* pp. 534-553.
6. Deeks, A.J. and Randolph, M.F., (1993), "Analytical Modeling Of Hammer Impact For Pile Driving", *Int. Journal of Num. and Anal. Methods in Geomechanics*, 17:279-302.

Method of Characteristics

7. Van Hamme et al, (1974), "Hydroblok and Improved Pile Driving Analysis", *De Ingeneur,* nr 8, Vol. 86.
8. Middendorp, P., (2004), "Thirty Years Experience with the Wave Equation Solution Based on the Method of Characteristics", *7th International Conference on the Application of Stress Wave Theory to Piles, K.L., Malaysia 2004.*
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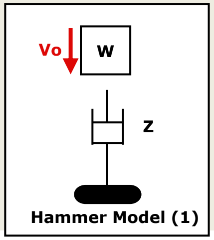
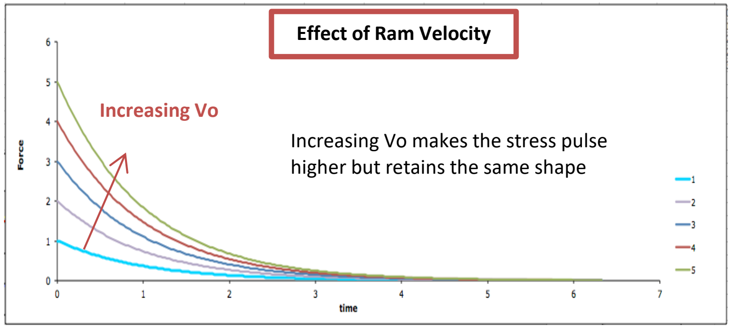
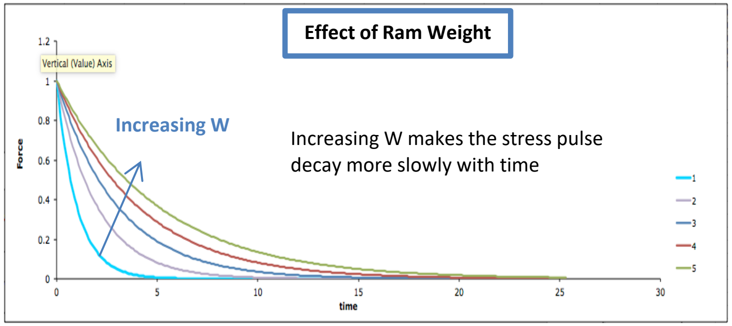
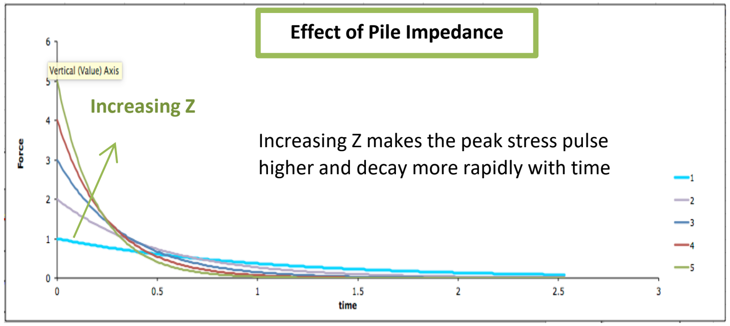


Hammer Model (1)







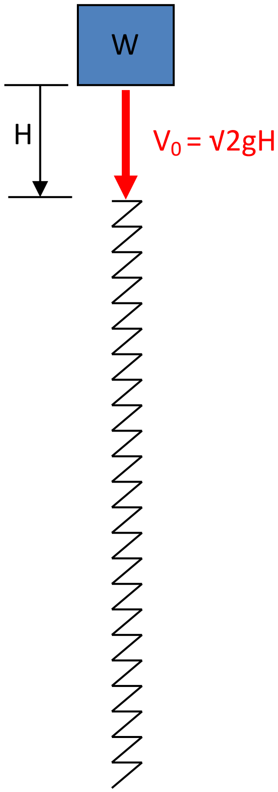
<u>**Direct impact of hammer ram on pile**</u>

This is the simplest model. The ram is assumed to be a lumped mass and the pile is assumed to act like a damper with impedance Z (infinitely long pile). The ram is assumed to impact directly on the top of the pile.

The initial velocity of the ram $V_0 = \sqrt{2gH}$ is transmitted to the top of the pile. This velocity creates a force with magnitude $F_0 = Z.V_0$ which then travels down the pile. The shape of the pulse is given by

$$F = Z.V_0 \,.\, e^{-[Z/W].t}$$

This formula is the key to understanding the major parameters that affect pile driving, i.e. W, V, and Z



Hammer Theoretical Model (2)

<u>Adding a cushion on top of the pile</u>

An additional spring with stiffness K1 is added to the top of the pile.

This model is still simple enough for theoretical formulas to be used.

However, it is too simplified to simulate the actual stress waves measured in PDA tests.

Ref:- Accuracy in Numerical Analysis for Pile Driving Dynamics – Deeks & Randolf, 1992

If the stiffness of the cushion is reduced to a finite value, then if $m_r > 4Z^2/k_c$ the force is given by the following equation (Randolph (1990)).

$$F(t) = Z\ v_o\ e^{-\frac{Z}{m_r}t}\ \frac{\sinh(\mu t)}{Z\mu/k_c} \qquad (2)$$

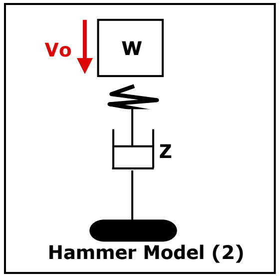
where $$\mu = \sqrt{\frac{k_c^2}{4Z^2} - \frac{k_c}{m_r}}$$

However, if $m_r \leq 4Z^2/k_c$, then the hammer will rebound from the pile at time $t_s = \pi/\mu'$, where μ' is given below. Prior to this time the force on the pile is given by equation (3), and after separation the force is zero (Deeks and Randolph (1992)).

$$F(t) = Z\ v_o\ e^{-\frac{Z}{m_r}t}\ \frac{\sin(\mu' t)}{Z\mu'/k_c} \qquad (3)$$

where $$\mu' = \sqrt{\frac{k_c}{m_r} - \frac{k_c^2}{4Z^2}}$$

Figure 2 shows the effect of the cushion stiffness on the shape of the forcing function. As the stiffness of the cushion decreases, the rise time decreases and the maximum force decreases. As the spring stiffness increases, the solution tends towards that given by equation (1), which corresponds to a zero rise time.



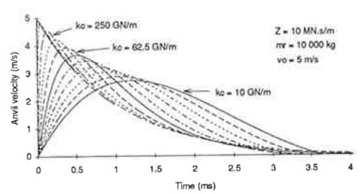
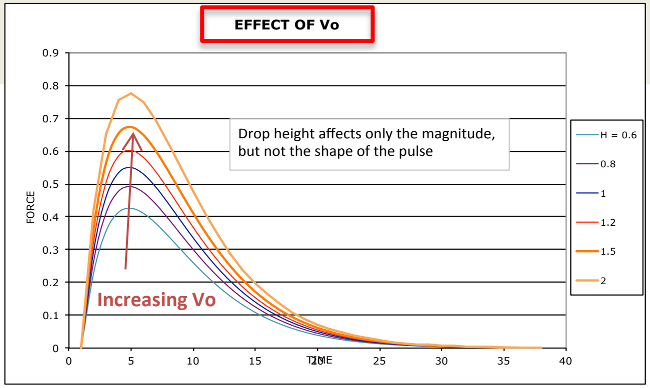
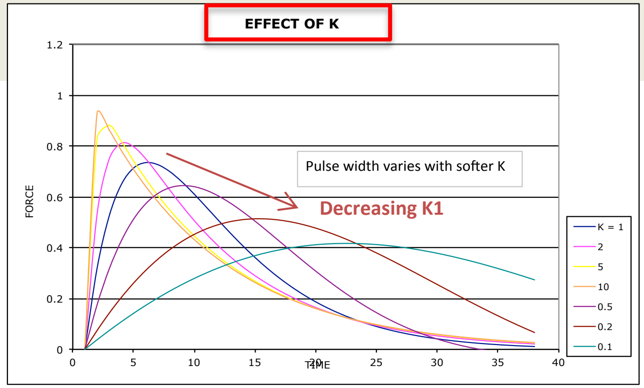
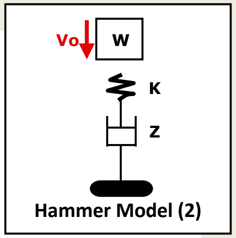


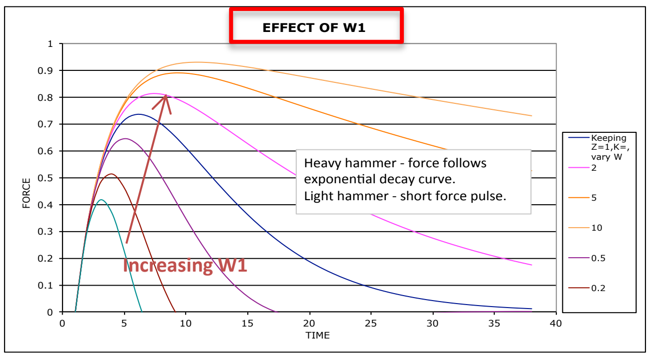
Fig.2 Variation of force with spring stiffness

Hammer Model (2)

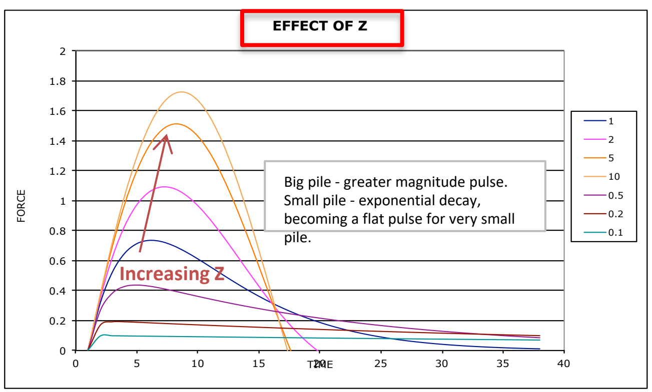








Hammer model [2] with a cushion on top of the pile begins to look more like the actual measured force pulse. Basically, the cushion rounds out the force peak and widens the pulse.



Theoretical Formulation - Hammer Model (3)

Ref:- Analytical Modeling of Hammer Impact for Pile Driving – Deeks & Randolph, 1993

APPENDIX I: SOLUTION OF RAM/CUSHION/ANVIL MODEL

The Laplace transform for the anvil velocity in the ram/cushion/anvil model is

$$L\dot{u}_a^* = \frac{\dfrac{k_c^*}{m_a^*}}{\dfrac{k_c^*}{m_a^*} + k_c^*\left(\dfrac{1}{m_a^*}+1\right)s + \dfrac{1}{m_a^*}s^2 + s^3} \quad (58)$$

Naming the coefficients of the cubic denominator

$$a_0 = \frac{k_c^*}{m_a^*}, \quad a_1 = k_c^*\left(\frac{1}{m_a^*}+1\right), \quad a_2 = \frac{1}{m_a^*} \quad (59)$$

equation (32) can be written as

$$L\dot{u}_a^* = \frac{a_0}{a_0 + a_1 s + a_2 s^2 + s^3} \quad (60)$$

Letting

$$\alpha = \frac{a_1 a_2}{6} - \frac{a_0}{2} - \frac{a_2^3}{27} \quad (61)$$

and

$$\beta = \sqrt{\left(\frac{a_1^3}{27} - \frac{a_1^2 a_2^2}{108} - \frac{a_0 a_1 a_2}{6} + \frac{a_0^2}{4} + \frac{a_0 a_2^3}{27}\right)} \quad (62)$$

if β^2 is greater than zero, the cubic denominator of equation (58) has one real root and two imaginary roots. The inverse transform can be found readily if the following substitutions are made:

$$b_1 = \frac{a_2}{3} - (\alpha+\beta)^{1/3} - (\alpha-\beta)^{1/3} \quad (63)$$

$$b_2 = \frac{a_2}{3} + \frac{1}{2}[(\alpha+\beta)^{1/3} + (\alpha-\beta)^{1/3}] \quad (64)$$

$$\omega = \frac{\sqrt{3}}{2}[(\alpha+\beta)^{1/3} - (\alpha-\beta)^{1/3}] \quad (65)$$

This allows the cubic denominator to be factorized, and equation (58) becomes

$$L\dot{u}_a^* = \frac{a_0}{(s+b_1)[(s+b_2)^2+\omega^2]} \quad (66)$$

which can be expanded to

$$L\dot{u}_a^* = \frac{a_0}{\omega^2+(b_2-b_1)^2}\left\{\frac{1}{s+b_1} - \left[\frac{s+b_2}{(s+b_2)^2+\omega^2} + \frac{b_2-b_1}{(s+b_2)^2+\omega^2}\right]\right\} \quad (67)$$

Performing the inverse Laplace transform, the anvil velocity can be found.

$$\dot{u}_a^* = \frac{a_0}{\omega^2+(b_2-b_1)^2}\left[e^{-b_1t^*} - e^{-b_2t^*}\left(\cos\omega t^* + \frac{b_2-b_1}{\omega}\sin\omega t^*\right)\right] \quad (68)$$

This solution can be written in a simpler, more convenient form by making

$$c_1 = b_1, \quad c_2 = b_2 - b_1, \quad \phi = \arctan\frac{c_2}{\omega}, \quad F_p = \frac{a_0}{\omega^2+c_2^2} \quad (69)$$

The anvil velocity and the force on the pile head are then

$$f_p^* = \dot{u}_a^* = F_p e^{-c_1t^*}\left(1 - e^{-c_2t^*}\frac{\cos(\omega t^* - \phi)}{\cos\phi}\right) \quad (70)$$

The spring force can be found by using the equilibrium equation

$$f_s^* = m_a^*\ddot{u}_a^* + \dot{u}_a^* \quad (71)$$

Differentiating equation (70), substituting into equation (71), and simplifying, the spring force can be expressed as

$$f_s^* = F_s\left(e^{c_2t^*} - \frac{\cos(\omega t^* - \theta)}{\cos\theta}\right) \quad (72)$$

where

$$\theta = \arctan\left(\frac{c_2(a_2-c_1-c_2)-\omega^2}{\omega(a_2-c_1)}\right) \text{ and } F_s = \frac{a_0(a_2-c_1)}{\omega^2+c_2^2} \quad (73)$$

For most combinations of m_a^* and k_c^*, β^2 is greater than zero, and the solution presented above applies. For a small range of combinations, β^2 is less than or equal to zero, and a different solution must be used. When this is the case, the cubic denominator has three real roots, and equation (58) can be written in the following way:

$$L\dot{u}_a^* = \frac{a_0}{(s+b_1)(s+b_2)(s+b_3)} \quad (74)$$

$$b_1 = \frac{a_2}{3} + 2\sqrt{Q}\cos\frac{\theta}{3} \quad (75)$$

$$b_2 = \frac{a_2}{3} + 2\sqrt{Q}\cos\frac{\theta+2\pi}{3} \quad (76)$$

$$b_3 = \frac{a_2}{3} + 2\sqrt{Q}\cos\frac{\theta+4\pi}{3} \quad (77)$$

$$Q = \frac{a_2^2 - 3a_1}{9} \quad (78)$$

$$R = \frac{2a_2^3 - 9a_1a_2 + 27a_0}{54} \quad (79)$$

$$\theta = \arccos\left(-\frac{R}{\sqrt{Q^3}}\right) \quad (80)$$

Too Complicated !

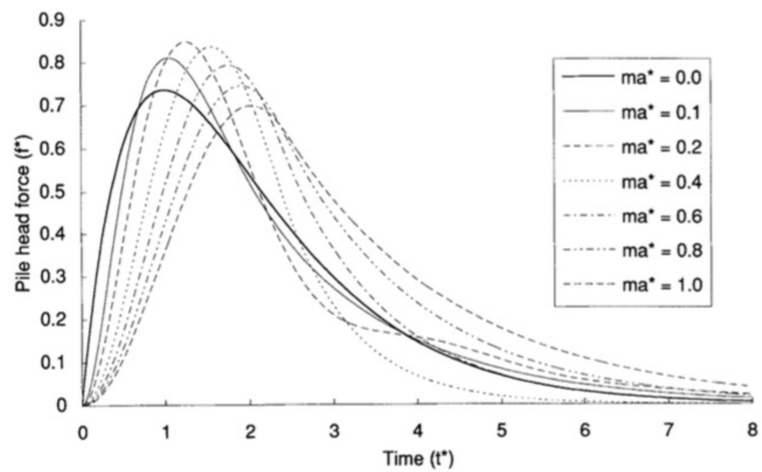


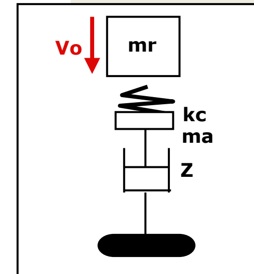
Figure 3. Variation of pile head force–time response with dimensionless anvil mass for a cushion stiffness of 4

$(b_2-b_1)(b_3-b_1)$, ... $a_2-b_1b_3-b_1$, ... $a_2-b_1b_2-b_3$

However, if two of the roots are the same, the solution is different again. When β is zero and b_3 is equal to b_2, the solution becomes

$$f_p^* = \dot{u}_a^* = F_p e^{-c_1t^*}[1 - e^{-c_2t^*}(1+b_2t^*)] \quad (87)$$

$$f_s^* = F_s e^{-c_1t^*}[1 - e^{-c_2t^*}(1+b_2A_st^*)] \quad (88)$$



Hammer Model (3)

PDA Notation

From PDA-W Manual of Operation[4]

1.14.6 Short Recommended List of Useful Result Quantities

FMX MAX FORCE
VMX MAX VELOCITY
DMX MAX DISPLACEMENT
DFN FINAL DISPLACEMENT
EMX MAX ENERGY
ETR ENERGY TRANSFER RATIO (= EMX / RATING) BPM BLOWS PER MINUTE
STK STROKE - O.E.DIESELS ONLY
CSX MAX COMPR-STRESS
CSI MAX INDIV.C-STRESS
CSB MAX TOE C-STRESS
TSX MAX TENSION STRESS
TSN MAX T-STRESS; WU ONLY
BTA INTEGRITY FACTOR
LTD LENGTH TO DAMAGE

PILE CAPACITY METHODS: RA2, [RMX, RSP] - JC; RX4..RX5..

Shaft Resistance: SFR - function of JC, or SF4, SF5, etc.

End Bearing: EBR - function of JC, or EB4, EB5, etc.